

## ANOVA between subjects

### *An alternative medical example for the same between-subjects design*

The data from a  $3 \times 3$  between-subjects design are shown in Table 2.4. This is a pilot study designed to investigate the efficacy of three drugs at three dosages in the preventative treatment of migraine. The selection criterion for 27 patients is at least 20 moderate or severe migraines in the past 12 months, based on an annual migraine diary in which occurrences of mild, moderate and severe migraine are recorded (a mild migraine is one that does not interfere with usual activities, a moderate migraine is one that inhibits but does not wholly prevent usual activities and a severe migraine is one that prevents all activities). Nine patients are randomly allocated to each of three drug types and, within each drug type, three are randomly allocated to each of three dose levels (low, medium and high). The factors are thus drug type (DRUG) with three levels (drug 1, drug 2 or drug 3) and drug DOSE with three levels (low, medium and high). The DV is reduction in the number of moderate or severe migraines within the following 12 months (SCORE). The 27 participants were randomly assigned to the 9 conditions, three to each condition, so each condition, or *cell in the table*, has three *replicates*. Each observation in the table represents one participant. As usual, if this were a real experiment, the number of participants per cell should be justified by a power analysis.

Table 2.4  
*Data from a factorial between-subjects design*

| drug type | drug dosage |        |      |
|-----------|-------------|--------|------|
|           | low         | medium | high |
| drug 1    | 6           | 12     | 18   |
|           | 4           | 9      | 25   |
|           | 8           | 10     | 22   |

|               |    |    |    |
|---------------|----|----|----|
| <b>drug 2</b> | 14 | 19 | 15 |
|               | 20 | 24 | 14 |
|               | 17 | 13 | 22 |
| <b>drug 3</b> | 9  | 14 | 10 |
|               | 14 | 12 | 12 |
|               | 12 | 19 | 20 |

### *A between-subjects design: setting it up in SPSS*

The SPSS datasheet should be arranged with each participant occupying a row and each variable occupying a column, so we need three columns for our variables SCORE, DRUG and DOSE. The order is not important but we list all the scores for low dose (coded 1), followed by all those for medium dose (2), and finally all those for high dose (3), making a column of length 27, one observation for each of the 27 participants. The next column gives the drug type (coded 1, 2 or 3) for each observation, so there are three 1s followed by three 2s and then three 3s, and the whole list of nine repeated twice more. The next column has nine 1s, then nine 2s and finally nine 3s. The order in which the variables are placed doesn't matter as long as it's the same for every participant. The first eleven rows of our datasheet appear as Table 2.5 (the full dataset can be found on the book website as med.anova.between.sav).

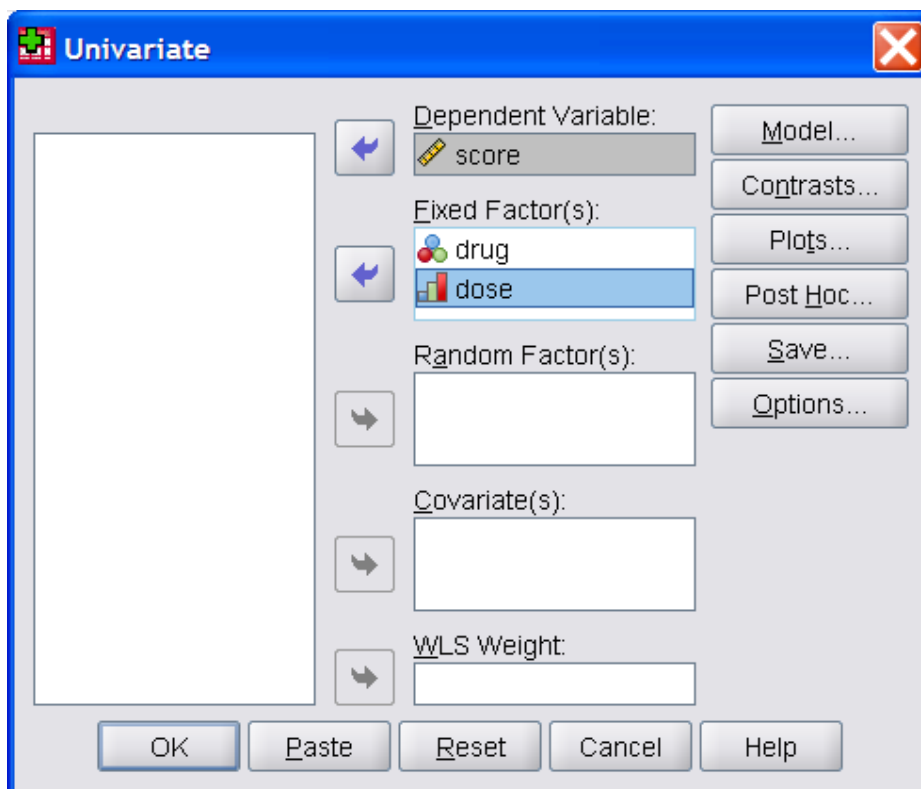
Table 2.5

*First few cases of data from a factorial between-subjects design set out for the SPSS datasheet (the full dataset can be found as med.anova.between.sav on the website)*

| <b>score</b> | <b>drug</b> | <b>dose</b> |
|--------------|-------------|-------------|
| 6            | 1           | 1           |
| 4            | 1           | 1           |
| 8            | 1           | 1           |
| 14           | 2           | 1           |
| 20           | 2           | 1           |
| 17           | 2           | 1           |
| 9            | 3           | 1           |
| 14           | 3           | 1           |
| 12           | 3           | 1           |
| 12           | 1           | 2           |
| 9            | 1           | 2           |

*A between-subjects design: requesting the analysis in SPSS*

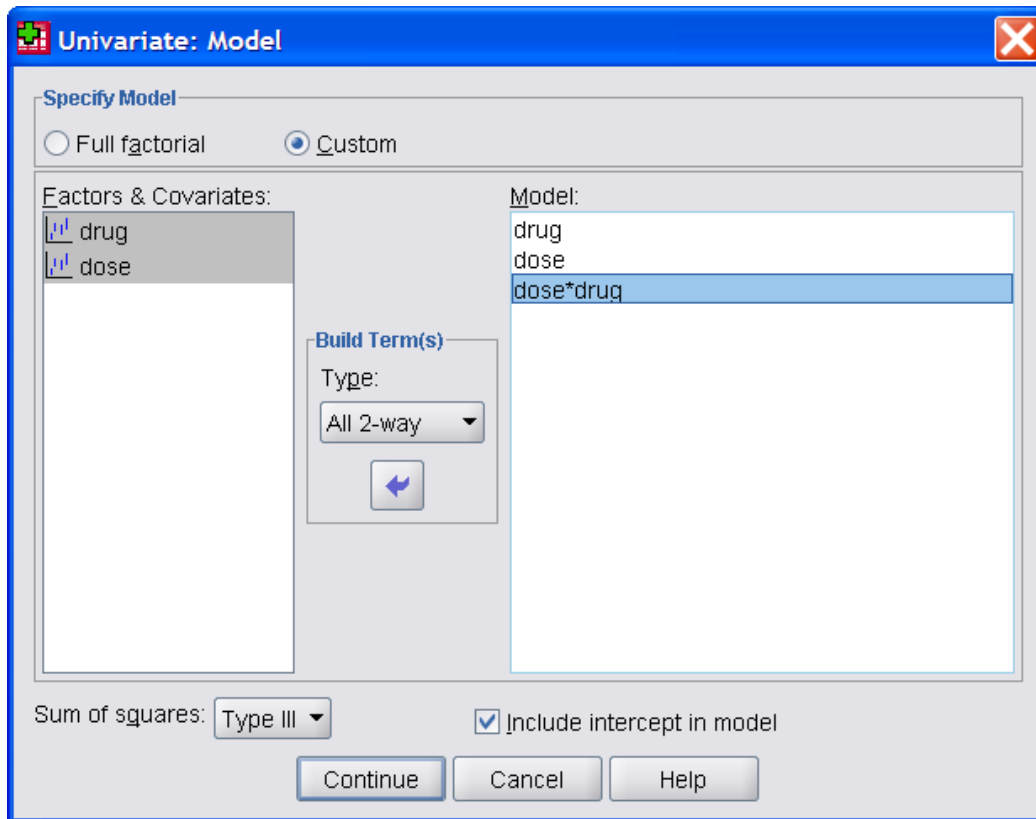
Once the datasheet is complete with its 27 rows and 3 columns, choose from the menu bar **Analyze**, then **General Linear Model**, then **Univariate**, to get SPSS Dialog Box 2.5. Select SCORE from the variable list and use the arrow to put it in the **Dependent Variable** box. Then put DRUG and DOSE in the **Fixed Factors** box, so the dialog box appears as shown. We shall not be considering random factors, and covariates are considered in Chapter 5, Analysis of Covariance (ANCOVA). The **WLS Weight** box allows you to apply weights to the observations, but again this is something we do not consider. If you click **OK** now you will get the ANOVA, but we will look at some of the extra information available from the other buttons.



*SPSS Dialog Box 2.5. Starting a factorial between-subjects ANOVA*

First click the **Model** button to get SPSS Dialog Box 2.6. The **Full factorial** radio button is the default, and we could accept this. This will include the main effect for each of our IVs and also the interaction between them. However, we will take this

opportunity to demonstrate how to build the required model yourself. It might be useful to be able to do this if, for example, the interaction turned out not to be significant and we decided to remove it in order to improve the power of the ANOVA to detect significant main effects. So we click the **Custom** radio button and build up the model terms ourselves. From the **Build Term(s)** menu select **Main effects**, then use the arrow to put both factors into the **Model** box. Then select either **Interaction** or **All 2-way** from the menu, select both factors and use the arrow to put the interaction in the **Model** box. Here, because we have only two factors, there is only one interaction between them, so **All 2-way** will just give us one two-way interaction, the same as if we selected **Interaction**. However, in an experiment with three factors, there would be three 2-way interactions (and one 3-way), so we could enter all of the 2-way interactions by selecting **All 2-way** and all three factors before clicking the arrow. If we want to enter just one of the 2-way interactions though (perhaps after finding that the other two are not significant), it's useful to have **Interaction** available on the menu to do this.

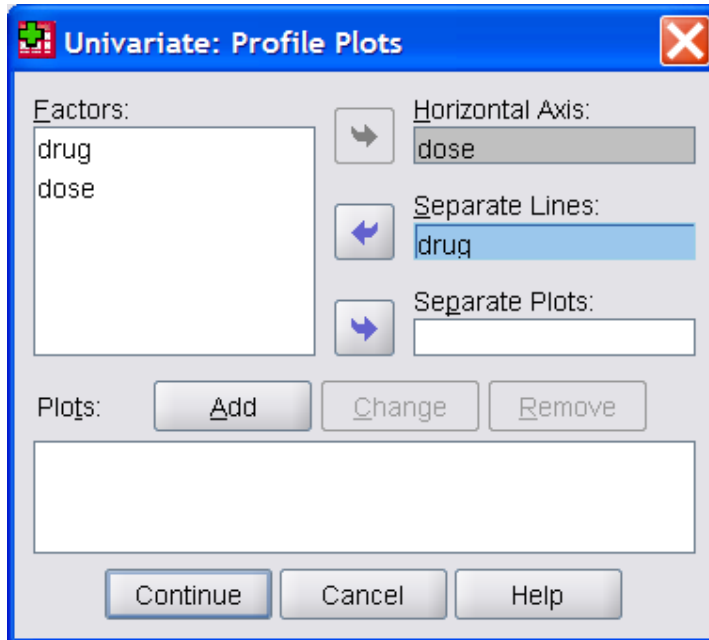


*SPSS Dialog Box 2.6. Specifying the model*

Near the bottom of the dialog box is a menu offering different choices for **Sum of squares**. Type III is the default, and almost always the one we want. Chapter 5 on ANCOVA gives a brief discussion of a situation where a different choice may be appropriate. Make sure **Include intercept in model** (the default) is ticked, otherwise we shall be assuming that the overall mean is zero.

Click **Continue** to return to SPSS Dialog Box 2.5, and click the **Plots** button to get SPSS Dialog Box 2.7. Select DOSE and use the arrow to put it in the **Horizontal Axis** box. Then put DRUG in the **Separate Lines** box. Click **Add** and DOSE\*DRUG appears in the **Plots** box. This will allow us to see a graph shown in SPSS Output 2.8. We could have reversed the roles of the two factors. In fact you can request both these plots if you wish, since when you click **Add** the **Horizontal Axis** and **Separate Lines**

boxes both empty, so you can now put DRUG in the **Horizontal Axis** box, DOSE in the **Separate Lines** box, and click **Add** again. Click **Continue** to return to SPSS Dialog Box 2.5.



*SPSS Dialog Box 2.7. Requesting a plot of the effects*

Now click the **Options** button to get a list of statistics for optional display. Since we have requested a visual display of the means in the plot, leave the **Display Means for** box empty. If we decide later that means for the nine conditions, or those for each of the three levels of DRUG and for each of the three levels of DOSE might be useful, we can always re-run the analysis and request them. We could also request a table of means for the interaction, which would give us a table of means for each of the 9 conditions in the experiment: remember that we had three observations (replicates) for each condition. But for now, omit all these. In the **Display** group click **Homogeneity tests**, which will provide a check on the assumption that variances are equal in all conditions. The **Residual plot** provides a check on the assumption of approximate normality so click this as well. The **Estimates of effect size** should be reported if our factors turn out to be significant, though you should be aware that SPSS provides

*partial* estimates of effect size (the proportion of factor plus error variance accounted for by the factor) and that may not be what you want. If you want full estimates of *eta squared* (that is the proportion of total variance accounted for by the factor), you will need to divide  $SS(\text{factor})$  by  $SS(\text{total})$  yourself. The Sums of Squares (SS) are given in SPSS ANOVA summary tables. The **Observed power** will be potentially useful for planning future experiments and should be reported in order to facilitate any future meta-analyses. So, we select **Estimates of effect size, Observed power, Homogeneity tests** and **Residual plot**, then click **Continue** to return to SPSS Dialog Box 2.5.

The **Save** button allows us to keep the values of the DV that are predicted by the model (they can be added to the datasheet). There are other statistics that can also be saved, but we will not use this facility. We also ignore the **Post hoc** button. We considered post hoc tests in connection with the one-way example. The same principles apply in relation to factorial designs but, if you want to use them there, you are advised to consult more detailed treatments of the topic (see Further reading at the end of the book).

We also introduced the use of the **Contrasts** option for follow-up tests in our one-way example. Again, the principles apply to factorial designs as well, but you probably need to read further before using **Contrasts** in factorial designs. In fact, we have found that it is often simpler to do these contrasts by hand, making use of some of the standard output from the SPSS analysis, but we are not going to embark further on this rather big topic. The buttons at the bottom are mostly self-explanatory (**Paste**

allows you to paste the SPSS commands and so use the command language). Press **OK** to get the analysis.

### *A between-subjects design: understanding the output*

The first table in the output (not shown here) just lists the factors, their levels and the number of observations at each level. Each of our factors has three levels, and there were 27 observations, 9 at each level of each factor, or 3 at each of the 9 combinations of factor levels. Next is a test for the equality of variances, a check on the homogeneity of variance assumption, shown as the first table in SPSS Output 2.6. Below the table is a reminder of the terms we included in our analysis. In our example,  $F(8,18)$  is only 1.069, and the probability of this (look at the Sig column) is well above 0.05, so the assumption of homogeneity of variance is satisfied.

Next comes the ANOVA table, the second in SPSS Output 2.6. The Intercept, or grand mean is significantly different from zero (look in the Sig column opposite Intercept) but this is rarely of any interest. Below that we see that the main effect of DRUG is significant at the 5% level ( $F(2,18) = 4.42, p = 0.027, <0.05$ ) with an effect size of partial  $\eta^2 = 0.33$  and retrospective (observed) power = 0.69. The main effect of DOSE is also significant at 5% ( $F(2,18) = 5.87, p = 0.011, <0.05$ ) with an effect size of partial  $\eta^2 = 0.40$  and power = 0.81. The interaction is just significant at the 1% level ( $F(4,18) = 4.59, p = 0.010$ ) with an effect size of partial  $\eta^2 = 0.51$  and power = 0.87.

**Levene's Test of Equality of Error Variances<sup>a</sup>**

| Dependent Variable: score |     |     |      |
|---------------------------|-----|-----|------|
| F                         | df1 | df2 | Sig. |
| 1.069                     | 8   | 18  | .426 |

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + drug + dose + drug \* dose

Tests of Between-Subjects Effects

Dependent Variable: score

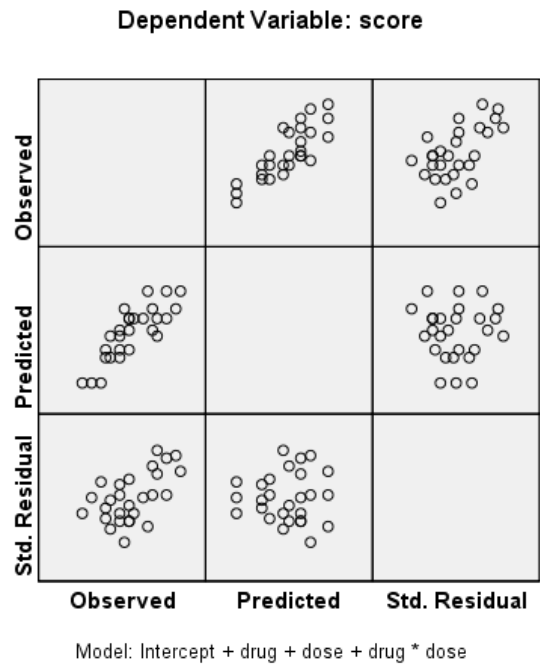
| Source          | Type III Sum of Squares | df | Mean Square | F       | Sig. | Partial Eta Squared | Noncent. Parameter | Observed Power <sup>b</sup> |
|-----------------|-------------------------|----|-------------|---------|------|---------------------|--------------------|-----------------------------|
| Corrected Model | 537.852 <sup>a</sup>    | 8  | 67.231      | 4.867   | .003 | .684                | 38.933             | .974                        |
| Intercept       | 5749.481                | 1  | 5749.481    | 416.182 | .000 | .959                | 416.182            | 1.000                       |
| drug            | 122.074                 | 2  | 61.037      | 4.418   | .027 | .329                | 8.836              | .685                        |
| dose            | 162.074                 | 2  | 81.037      | 5.866   | .011 | .395                | 11.732             | .811                        |
| drug * dose     | 253.704                 | 4  | 63.426      | 4.591   | .010 | .505                | 18.365             | .870                        |
| Error           | 248.667                 | 18 | 13.815      |         |      |                     |                    |                             |
| Total           | 6536.000                | 27 |             |         |      |                     |                    |                             |
| Corrected Total | 786.519                 | 26 |             |         |      |                     |                    |                             |

a. R Squared = .684 (Adjusted R Squared = .543)

b. Computed using alpha = .05

*SPSS Output 2.6. The test for the homogeneity of variance assumption and the ANOVA summary table*

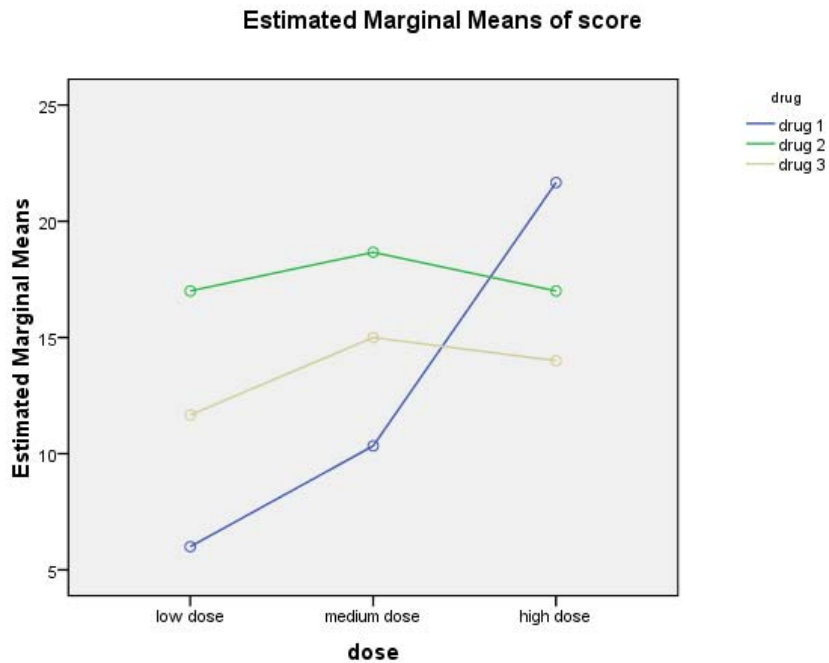
Since the interaction is significant, as we saw earlier we must interpret the main effects with care. We will return to this after we have looked at the remainder of the output. We consider next our Residual plot, shown as SPSS Output 2.7. A reminder of the model used is at the bottom. The useful plot from here is in the centre of the bottom row. This one shows the predicted values of the DV from the model on the x-axis, and the residuals on the y-axis. The residual of an observation is the difference between the observation and the value predicted by the model. Here the residuals have been standardized so they have a mean of zero and a standard deviation of 1. If our Normality assumption is correct, the standardized residuals are standard Normal random variables, and this plot should show a shapeless cloud of points. Our plot is indeed a shapeless cloud of points and we can take it that for our data, the Normality assumption is satisfied.



*SPSS Output 2.7. Residual plots*

The graph at centre left shows the predicted versus the observed values: a perfect fit would give a straight line, but of course there are always bits of random variation. The three graphs at upper right are just mirror images of those at lower left. The graph of standardized residuals versus observed values is of no interest since the residuals are always correlated with the observed values.

So we consider next our plot, which appears at the end of the output, and is shown as SPSS Output 2.8. Here we see that drugs 2 and 3 show similar patterns, an increase in the SCORE when we increase the DOSE from low to medium, and a slight decrease when DOSE is increased again to high. The drug 2 scores between about 3 and 5 higher than drug 3 at every level of DOSE. The pattern for drug 1 is quite different, with the SCORE being very much higher at high dose than at the medium dose. This difference in patterns will account for the significance of the interaction.



*SPSS Output 2.8. A plot of the means*

*A between-subjects design: simple effects following a significant interaction*

As we have already mentioned, because the interaction is significant, we really need to compare DRUGs at each level of DOSE; that is, we need to examine the simple effects of DRUG. The simplest way to do this is to split the data into three, a set for each level of DOSE. Then we carry out a one-way ANOVA on each of the three datasets. To split the data into three sets, while in the SPSS datasheet, select **Data** from the menu bar and then **Split File**. Click on the radio button **Organize output by groups** and use the arrow to move DOSE into the **Groups Based on:** box. Check that **Sort the file by grouping variables** is selected and click **OK**. Then proceed to request a one-way analysis just as in the previous one-way example. In the dialog box that is like SPSS Dialog Box 2.2, move SCORE into the **Dependent List** box and DRUG into the **Factor** box and click **OK**. SPSS will do three one-way analyses, one for each

level of DOSE. The results, an ANOVA of SCORE for each level of DOSE, are shown in SPSS Output 2.9.

**ANOVA<sup>a</sup>**

| score          |                |    |             |        |      |
|----------------|----------------|----|-------------|--------|------|
|                | Sum of Squares | df | Mean Square | F      | Sig. |
| Between Groups | 181.556        | 2  | 90.778      | 14.086 | .005 |
| Within Groups  | 38.667         | 6  | 6.444       |        |      |
| Total          | 220.222        | 8  |             |        |      |

a. dose = low dose

**ANOVA<sup>a</sup>**

| score          |                |    |             |       |      |
|----------------|----------------|----|-------------|-------|------|
|                | Sum of Squares | df | Mean Square | F     | Sig. |
| Between Groups | 104.667        | 2  | 52.333      | 3.438 | .101 |
| Within Groups  | 91.333         | 6  | 15.222      |       |      |
| Total          | 196.000        | 8  |             |       |      |

a. dose = medium dose

**ANOVA<sup>a</sup>**

| score          |                |    |             |       |      |
|----------------|----------------|----|-------------|-------|------|
|                | Sum of Squares | df | Mean Square | F     | Sig. |
| Between Groups | 89.556         | 2  | 44.778      | 2.264 | .185 |
| Within Groups  | 118.667        | 6  | 19.778      |       |      |
| Total          | 208.222        | 8  |             |       |      |

a. dose = high dose

*SPSS Output 2.9. One-way ANOVAs to test for simple effects following a significant interaction*

These results suggest that only the simple effect of DRUG at the low level of DOSE is significant ( $F(2,6) = 14.086, p < 0.01$ ). However, it is legitimate to use all of the data (i.e., from all levels of DOSE) to get a better estimate of the error (within groups) variance, provided that variances are homogeneous across conditions. As this is a reasonable assumption in this case, we will adopt that strategy. To do this, replace the within groups MS in each one-way table with the within groups MS (13.815, labeled Error in SPSS Output 2.6) from the main 3 x 3 ANOVA. Then use that value in the formula  $F = MS(\text{drug})/MS(\text{Error from main ANOVA})$  to obtain a new  $F$  value for each simple effect. The values are:  $F(\text{low dose}) = 6.57$ ,  $F(\text{medium dose}) = 3.86$ ,  $F(\text{high dose}) = 3.24$ , all with 2 and 18  $dfs$  (from the main ANOVA table). We refer to tables for the  $F$  distribution and find that, with 2 and 18  $dfs$ , the critical value with

alpha at 0.05 is  $F_{\text{crit}} = 3.55$  and that with alpha at 0.01 is  $F_{\text{crit}} = 6.01$ . So, we find the simple effect of drug at the low dose to be significant at  $p < 0.01$  and that at the medium dose to be significant at  $p < 0.05$ . So, that little bit of extra work was quite worthwhile.

You may note that, having found significant simple effects of DRUG, we may still want to find out whether pairs of drug levels differed from one another at the given level of DOSE. This is a question that can be addressed with a Tukey post hoc test, for example. Finally, it would be perfectly feasible to examine the simple effects of DOSE at each level of DRUG as well as, or instead of, the simple effects of DRUG at each level of DOSE. Which we choose to look at is just a matter of the focus of our interest; in this case probably on DRUG more so than on DOSE.